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# Some considerations regarding the determination, by historical data, of the elimination probalilities from a collectivity for different reasons

***di***
***Maria Sole Staffa***

**Abstract.** The collectivity analysis characterized by the belonging to a certain institution continues to be, in the actuarial literature, an object of great interest. It is sufficient to consider a pension fund subscribers and/or the members of a certain company and the interest in the elimination study from these communities.
In this work are resumed some considerations regarding the pure and relative probabilities of elimination from a collectivity exposed to more exit reasons, well known subject, in literature since a long time, deepening and specifying some aspects related to the determination of these probabilities from historical raw data. In particular, the present work purpose is to compare the annual exit probabilities because of death or for other reasons from a collectivity acquired through the quantification of the exposed to the risk according to the current definition in literature with the probabilities acquired through a monthly analysis of the collectivity accounting.



## 1. Foreword.

Supposing to have, at a certain date, a group of subscribers, for instance, to a pension fund, classified by age and eventually for subscribing seniority to the fund. It should be supposed, therefore, to observe in a subsequent statistic period that from this collectivity has been eliminated a certain number of people because of each one of the possible elimination reasons (e.g. death, disability, surrender, transfer to another fund,). Let us suppose performing these observations for instance for 3 or 5 years and that from these data we want to determine the probabilities to be eliminated for each of the reasons. These probabilities may be used to study the future evolution of the collectivity itself and therefore to determine for each of the considered collectivity life future years, e.g., average residual lifespan.
Of course, this analysis can be performed in relation to a collectivity at a certain date and until the exhaustion of the same or in presence of new incoming people.
In the collectivity study representing a pension fund, due to the subscribers

number, frequently “low”, from the statistical point of view, usually it is made recourse to communities with characteristics similar to those on object. In this work, we try to get “something” from the real collectivity even if of small numerousness.

## 2. Trend analysis of a collectivity: absolute and relative probabilities

It must be considered a close collectivity K subject to only one elimination reason A. It is $l(t)$ the variable which represents the numerousness of K at the time changing t with $l(0)$ which expresses the numerousness of k at the time 0.
It must be indicated with $m_A(t)$ the number of eliminated people from the collectivity during the period between $0$ and $t$

$$m_A(t) = l(0) - l(t)$$

It must be supposed that the function $l(t)$ enclosed is continuous and where needed derivable and it is indicated with $\mu_A(t)$ elimination instantaneous rate.
Then the demographic movement between $0$ and $t$ can be expressed by the following

$$l(t) = l(0) - \int_0^t l(u)\mu_A(u)du$$

having calculated the eliminated people from the collectivity through the elimination instantaneous rate.
It is defined the elimination rate from the collectivity in the period $(0,t)$ the ratio between the eliminated people number and the collectivity numerousness in 0

$$q(0,t) = \frac{l(0)-l(t)}{l(0)} = \frac{1}{l(0)}\int_0^t l(u)\mu_A(u)du \qquad (1)$$

In addition, this elimination rate indicates the probability that an individual belonging, at t=0, to the collectivity is eliminated within the time t.
The same probability can be expressed in function of the only instantaneous interest rate, starting from the definition of the instantaneous rate

$$\mu_A(t) = -\frac{d}{dt}\ln l(t) \qquad (2)$$

and integrating both members between 0 and t,

$$\int_0^t \mu_A(u)\,du = -\ln\frac{l(t)}{l(0)}$$

$$\exp\left(-\int_0^t \mu_A(u)\,du\right) = \frac{l(t)}{l(0)}$$

$$l(t) = l(0)e^{-\int_0^t \mu_A(u)du} \qquad (3)$$

The elimination probability, therefore, can be expressed also through

$$q(0,t) = \frac{l(0) - l(t)}{l(0)} = 1 - e^{-\int_0^t \mu_A(u)du}$$

Considering, then, a generic period ($t$, s) with $s$ > t it is possible to rewrite the (1) e (3):

$$q(t,s)\frac{1}{l(t)}\int_t^s l(u)\mu_A(u)du$$

$$q(t,s) = 1 - e^{-\int_t^s \mu_A(u)du} \qquad (3')$$

It must be considered, now, the case of a collectivity exposed to more elimination cases (supposing two, A and B): the individuals belonging to the collectivity are exposed simultaneously to the two risks, which cause their exit. It is supposed, however, that the two events cannot occur at the same time: an individual, in fact, comes out from the collectivity or for the reason A or for the reason B (the analysis where the two risks can be simultaneously exit reason can be performed considering 3 reasons A,B A and B).
The collectivity must be observed during the period $(0,t)$ and it is indicated with $m_{A(B)}(t)$ the number of individuals who came out from the collectivity for the reason A between the date 0 and the date t if the collectivity is exposed also to the elimination reason B. In analogous manner it is to be defined $m_{B(A)}(t)$.
The individuals eliminated during the period $(0,t)$ are

$$l(0) - l(t) = m_{A(B)}(t) + m_{B(A)}(t)$$

It is defined the elimination relative probability for to the reason A during the period $(t,s)$ even in presence of the reason B

$$q_{A(B)}(t,s) = \frac{m_{A(B)}(s) - m_{A(B)}(t)}{l\,(t)}$$

Having calculated by $m_{A(B)}(s) - m_{A(B)}(t)$ the number of individuals eliminated during the period $(t,s)$ for the only reason A in the case when the collectivity should be exposed also to the reason B.
In analogous manner the elimination relative possibility for the reason B during the period $(t,s)$ in presence of the reason A is

$$q_{B(A)}(t,s) = \frac{m_{B(A)}(s) - m_{B(A)}(t)}{l(t)}$$

With these positions, it comes that the probability of total elimination during the period *(t, s)* is the sum of the two relative probabilities always related to the same period.

$$q(t,s) = \frac{l(t) - l(s)}{l(t)} =$$

$$= \frac{l(0) - m_{A(B)}(t) - m_{B(A)}(t) - [l(0) - m_{A(B)}(s) - m_{B(A)}(s)]}{l(t)} =$$

$$= \frac{[m_{A(B)}(s) - m_{A(B)}(t)] + [m_{B(A)}(s) - m_{B(A)}(t)]}{l(t)} =$$

$$= q_{A(B)}(t,s) + q_{B(A)}(t,s)$$

By analogy with the case of the collectivity exposed to only one elimination reason, seen in the previous paragraph, if the functions $m_{A(B)}(t)$ e $m_{B(A)}(t)$ are regular enough it is possible to suppose the existence of an elimination instantaneous rate for the reason A even in presence of B

$$\mu_{A(B)}(t) = \frac{m'_{A(B)}(t)}{l(t)}$$

and the existence of an instantaneous rate function of elimination for the reason B also in presence of A

$$\mu_{B(A)}(t) = \frac{m'_{B(A)}(t)}{l(t)}$$

through which it is possible to determine the number of individuals who left the collectivity for each reason during the period (0,t)

$$m_{A(B)}(t) = \int_0^t l(u)\mu_{A(B)}(u)du$$

$$m_{B(A)}(t) = \int_0^t l(u)\mu_{B(A)}(u)du$$

The totally eliminated people, therefore, can be "counted" through the elimination instantaneous rates, that is

$$l(0) - l(t) = \int_0^t l(u)\mu_{A(B)}(u)du + \int_0^t l(u)\mu_{B(A)}(u)du =$$

$$= \int_0^t l(u)[\mu_{A(B)}(u) + \mu_{B(A)}(u)]du \quad (4)$$

According to a setting present in literature [2], it seems correct to consider the instantaneous related rates of elimination as independent from the co-presence of other elimination reasons. According to this setting, so, $\mu_A(\quad)$ is the expression rate of the reason A which leads to the exit and not as elimination rate for the reason A. It results obvious, then, on this point of view, that the "expression" rate of the reason A remains constant both in the case when the event "expressing of A" is the only reason, and in case that there is another event which causes the exit from the collectivity. Therefore, it seems correct to consider

$$\mu_A(\ ) = \mu_{A(B)}(\ ) \text{ e } \mu_B(\ ) = \mu_{B(A)}(\ ).$$

By these explanations, the number of individuals totally eliminated during the period (0.t) is given by

$$l(0) - l(t) =$$
$$= \int_0^t l(u)\mu_A(u)du + \int_0^t l(u)\mu_B(u)du =$$
$$= \int_0^t l(u)[\mu_A(u) + \mu_B(u)]du \qquad (5)$$

Coming back to the relative probabilities of elimination in the period (t, s) it is possible to rewrite them in the following manner:

$$q_{A(B)}(t,s) = \frac{m_{A(B)}(s) - m_{A(B)}(t)}{l(t)}$$
$$= \frac{\int_0^s l(u)\mu_{A(B)}(u)du - \int_0^t l(u)\mu_{A(B)}(u)du}{l(t)}$$
$$= \frac{1}{l(t)}\int_t^s l(u)\mu_{A(B)}(u)du = \frac{1}{l(t)}\int_t^s l(u)\mu_A(u)du \qquad (6)$$
$$q_{B(A)}(t,s) = \frac{m_{B(A)}(s) - m_{B(A)}(t)}{l(t)}$$
$$= \frac{\int_0^s l(u)\mu_{B(A)}(u)du - \int_0^t l(u)\mu_{B(A)}(u)du}{l(t)}$$
$$= \frac{1}{l(t)}\int_t^s l(u)\mu_{B(A)}(u)\,du = \frac{1}{l(t)}\int_t^s l(u)\mu_B(u)du$$

It is to be observed, however, that the probability (6) and the probability (3') are different between them. It is possible to demonstrate that between them there is the following relation [3]:

$$q_{A(B)}(t,s) = q_A(t,s) - \frac{1}{\mathrm{l(t)}}\int_{\mathrm{t}}^{s} \mathrm{l(u)}\mu_{\mathrm{B}}(\mathrm{u})q_A(t,u)du$$

The probability $q_A(t,s)$ for the reason A and the analogous for the reason B are called Pure (or absolute) Probabilities because they depend only on the instantaneous rate of elimination $\mu_A(\ )$, while $q_{A(B)}(t,s)$ are considered as relative because they depend also on $\mu_B(\ )$. In order to understand better this aspect, it is possible to analyze the collectivity which initially was exposed to 2 exit reasons like if the occurrence of one of the two events stops cause the exit from the collectivity: the pure probabilities of

elimination from the collectivity are acquired "purifying" the dynamics, observed in presence of the two reasons, by the effect of one of the two reasons.
Between the total probabilities of elimination and the pure probabilities there is, in addition, the following relation pertaining to Karup:

$$q(t,s) = q_A(t,s)+q_B(t,s)-q_A(t,s)q_B(t,s)$$

so

$$q(t,s) = \frac{l(t) - l(s)}{l(t)} = 1 - e^{\int_t^s [\mu_A(u)+\mu_B(u)]du}$$

$$1 - q(t,s) = e^{-\int_t^s [\mu_A(u)]du} e^{-\int_t^s [\mu_B(u)]du} =$$
$$[1 - q_A(t,s)][1 - q_B(t,s)] = 1 - q_A(t,s) - q_B(t,s) - q_A(t,s)q_B(t,s)$$

Made these considerations on the theoretical setting of the absolute and relative probabilities it must be done an important observation.
In the Social Insurances Technique it is necessary to use elimination probabilities from the collectivity, object of evaluation, which, of course, for what said so far, must be relative.
The determination of these probabilities comes from, as previously said, raw rates determined also upon historical data of the collectivity.
A common procedure, used in the practice of the Insurance Statistics is the one to use, for the death probabilities, a comparison with the death probabilities of the General Population in order to determine of how much must be varied the death probability of the general population to be used in the collectivity studied,
So it raises the problem that it is needed to compare the data of the elimination rates because of collectivity death (determined in relative manner) with the data coming from the general population, whose study has been performed considering only a single reason of elimination, the death exactly, because of this there is a logical leap to be justified.
It must be considered that because it is impossible to determine the absolute probabilities regarding the collectivity examined because it is impossible to follow the individuals eliminated from the collectivity for reasons different from the death, it must be justified on the approximation that the current procedure bears as inherent in itself.
In any case, the eliminations from the collectivity for the ages achievement and/or maximum seniority are different from the elimination for migrations (in terms of age) which are re-included in the death probabilities of the general population.

### 3. An empirical application – recurring equation

In this paragraph are verified the observations performed regarding the

various elimination probabilities, in a real collectivity publishing an aggregate mortality table through the death rates of a collectivity of homogeneous individuals (employees by an industrial enterprise) observing it daily for a period of three years precisely from December 31st. 2004 (time $t_0$) up to December 31st. 2007 (time $t_1$).
In the analysis we are preparing to perform through the collectivity observation, are determined the raw rates of elimination. These values are considered reliable for the probabilities on the sense of what reported in the footnote [4].
For all the individuals belonging to the collectivity, have been recorded the birth date, the date of entry in the collectivity and the date of possible exit storing their reason.
It has to be indicated by $N_{t_0}$ the numerousness in the time $t_0$ and by $N_{t_1}$ the numerousness in $t_1$.
It must be indicated, then by $i_x$ the individuals belonging to the collectivity in $t_0$ with age included between $x$ (excluded) and $x+1$(included), by $x$ whole number, and by $e_x$ the individuals belonging to the collectivity in $t_1$, that means at the end of the observation period, always of age included between $x$ (excluded) and $x+1$(included).
Are $x_m$ and $x_M$ the minimum and maximum age of the individuals belonging to the collectivity.
It is verified, in obvious manner, that

$$\sum_{x_m}^{x_M} i_x = N_{t_0} \text{ e } \qquad \sum_{x_m}^{x_M} e_x = N_{t_1}$$

It is supposed, initially, that the collectivity is closed to new admissions and that the only reason of exit is the death.
It is, then, $U(t_0,t_1)$ the number of individuals who left the collectivity since the time $t_0$ up to the time $t_1$ , that is

$$U(t_0,t_1) = N_{t_0} - N_{t_1}$$

Is $\vartheta_x$ the number of deaths of individuals originally belonging to the collectivity of age included between $x$ (excluded) and $x+1$(included) during the observation period.

$$\sum_{x_m}^{x_M} \vartheta_x = U(t_0,t_1)$$

It must be considered now the particular observation period $(t_0,t_1)$ with $t_1 = t_0 + 1$ and it must be indicated by $l_x^0$ the number of individuals who in $t_0$ has the exact age x: with the locations made it is possible to determine the exit raw rate for age $q_x$ between the ages $x$ and $x+1$, with

$$\vartheta_x = q_x \cdot l_x^0$$

Usually, the collectivity should be open to new incomings and there are other exit reasons over the death. Is $n_x$ the number of next entries in the collectivity of age included between $x$ (excluded) and $x+1$(included) during the observation period and $\omega_x$ the number of entries eliminated from the collectivity of age included between $x$ (excluded) and $x+1$(included).
It must be indicated by $l_x$ the number of individuals reaching the exact age $x$ during the observation period.
It is easy to verify the following recurrent relation

$$l_{x+1} = l_x + i_x + n_x - \vartheta_x - \omega_x - e_x \tag{7}$$

which expresses the movement of an open collectivity of policy holders exposed to more elimination reasons, having performed the "accounting" of the collectivity under examination distinguishing two exit reasons, the death and other reasons. .

## 4. Exit rates – Exposed to the risk

The traditional approach present in literature to calculate the exit probabilities for death and for other reasons it's necessary quantify the so-called Exposed to the elimination risk, indicated by $E_x$ for all ages:in that way it's possible to determine the probability $q_x$ as relationship among the statistic data available and the exposed to the risk.

When, for instance, are calculated with y $\vartheta_x$ the people who left for death aged between *x* (excluded) and $x+1$(included) it is to be considered that the observation period of these ones could have been different. First of all, there are, so , individuals present at the time $t_0$ and that during that period have exactly $x$ years, individuals that in that moment have $x+r$ years (with r included between 0 and 1) and, therefore have been observed in the class x only for a period lasting like $(x-1)-(x-r)=(r-1)$. So, indicating with ${}_{1-r}q_{x+r}$ the death probability of an individual of age $x+r$ to die within $1-r$ years that is within the age $x+1$: given the observations performed, it is possible to "quantify" the deaths through the following relation:

$$\vartheta_x = l_x\, q_x + \sum_r\ {}_{1-r}q_{x+r}\, i(x+r) \tag{8}$$

having indicated with $(x+r)$ the individuals who at the beginning of the observation period have the exact age $x+r$ so that

$$i_x = \sum_r i(x+r)$$

In addition, there are individuals who at the end of the observation period have $+r$ years (indicated with $(x+r)$ ) and therefore have been observed only by $x$ to $x+r$. It is necessary, therefore, to consider in correct way their contribution by subtracting an additional term to the (8).

$$\vartheta_x = l_x\, q_x + \sum_r {}_{1-r}q_{x+r}\, i(x+r) - \sum_r {}_{1-r}q_{x+r}\, e(x+r)$$

Besides, if the collectivity is open to new entries it is necessary to consider that between the $\vartheta_x$ there could be the individuals who entered the collectivity in a second time with exact age $x$ or with age $x+r$, and therefore

$$\vartheta_x = l_x\, q_x + \sum_r {}_{1-r}q_{x+r}\, i(x+r) + \sum_r {}_{1-r}q_{x+r}\, n(x+r) + \\ - \sum_r {}_{1-r}q_{x+r}\, e(x+r)$$

Besides, envisaging the possibility of exit for reasons different from the death which takes place in exact age $x+r$, then observed only with $x$ to $x+r$, it is necessary to consider also these and therefore

$$\vartheta_x = l_x\, q_x + \sum_r {}_{1-r}q_{x+r}\, i(x+r) + \sum_r {}_{1-r}q_{x+r}\, n(x+r) + \\ - \sum_r {}_{1-r}q_{x+r}\, \omega(x+r) - \sum_r {}_{1-r}q_{x+r}\, e(x+r)$$

To be able to locate what we are going to indicate with Exposed to the Risk, it is necessary to make an assumption on the probability ${}_{1-r}q_{x+r}$. It is possible to use a simple approximate expression for the probability ${}_{1-r}q_{x+r}$ supplied with the Balducci [5] assumption, of normal use in the statistical applications, regarding the adoption of which we will deal later on, for which

$${}_{1-r}q_{x+r} = (1-r)q_x$$

It is possible, therefore, to rewrite the number of the individuals who left for death of age included between $x$ (excluded) and $x+1$(included) in the following way

$$\vartheta_x = q_x \begin{pmatrix} l_x + \sum_r (1-r)[\, i(x+r) + n(x+r)] + \\ -\sum_r (1-r)[\, \omega(x+r) + e(x+r)] \end{pmatrix}$$

In which the "number" (not necessarily whole) of individuals calculated within the round brackets quantifies the so-called Exposed to the risk.

$$E_x = l_x + \sum_r (1-r)[\, i(x+r) + n(x+r)] + -\sum_r (1-r)[\, \omega(x+r) + e(x+r)]$$

It is observed, besides, that the number of Exposed to the Risk is included because it seems correct that each individual supplies with a contribution to the pertaining class equal to his "presence" period of time and, therefore, of exposure to the risk.
The probabilities of exit for death, therefore, can be determined through the following relation

$$\vartheta_x = q_x \cdot E_x$$

Similarly, it is possible to quantify also the exit probabilities for reasons different from the death $q_x^{\omega}$, calculating the "corrected people" exposed to the risk $E_x^{\omega}$

$$\omega_x = q_x^{\omega} \cdot E_x^{\omega}$$

It is observed that the analysis of the exit probabilities for death or for other reasons based on the quantification of the Exposed to the risk, could involve some problem of substantial character. It is right to precise, in fact, that with $E_x$ it is calculated the average number of the Exposed to the risk having adopted assumptions on the distribution of exit probability and in particular on ${}_{1-r}q_{x+r}$: with this assumption sometimes the number of who effectively left used in the above described formula could be greater than the exposed people number. In the empirical application this problems have been resolved by imposing suitable restrictions to the unknown exit probabilities.

## 5. An alternative approach: monthly analysis of death rates

It is presented now a different methodology to acquire from the observation of a collectivity the rates of exit for death and for other reasons.
It must be considered the open collectivity introduced in the previous

paragraph and are to be determined $l_{x+r}, i_{x+r}, n_{x+r}, \vartheta_{x+r}, \omega_{x+r}, e_{x+r}$ the sizes relevant to the collectivity at the age x + r with x number of years and r=1/12,...,12/12 year fraction. It is to be verified that the sizes above identified also in this case satisfy the following relation (7).
Through the amounts identified it is possible to determine the actual ${}_{1/12}q_{x+r}$ that is the probabilities of a person of age x + r years to exit for death within 1 month, through the following relation

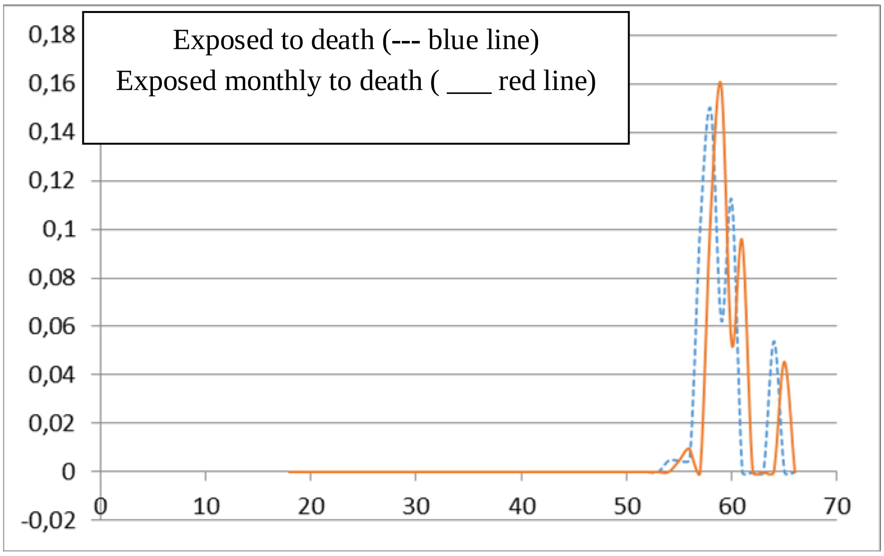
$$\vartheta_{x+r} = {}_{1/12}q_{x+r} \cdot l_{x+r}$$

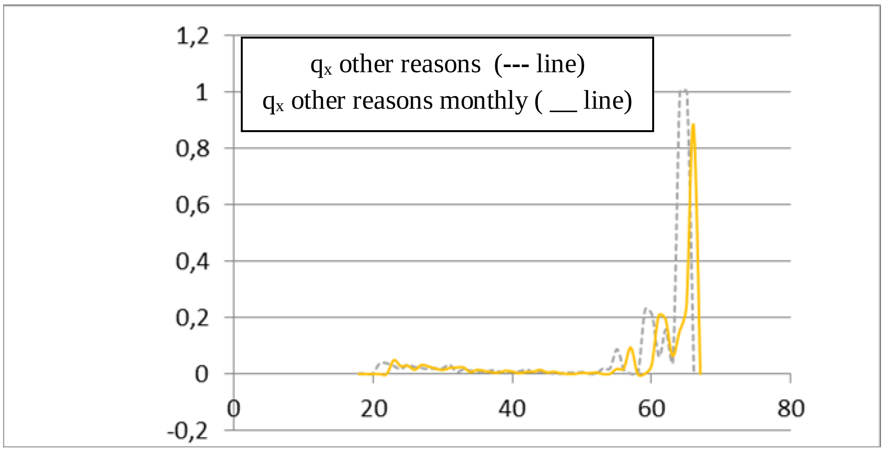
and in the same manner the ${}_{1/12}q^{\omega}_{x+r}$.

Therefore, being known these properties, it is possible to acquire the annual probabilities of elimination for death $\bar{q}_x$ (and for the other reasons $\bar{q}^{\omega}_x$ ) through [6] the

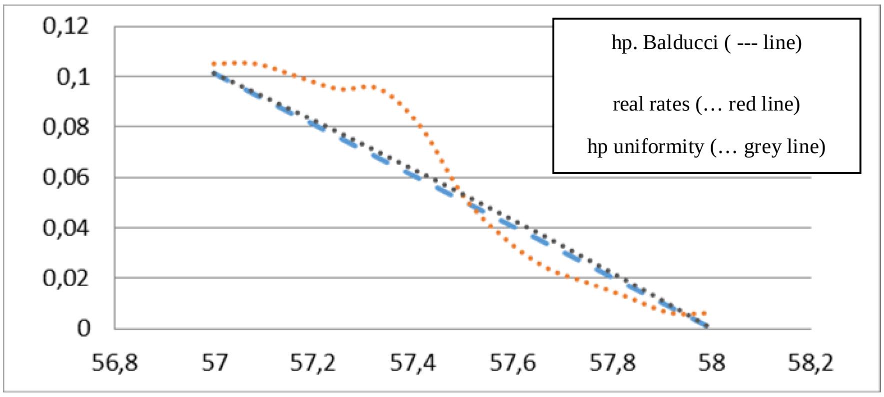
$$\bar{q}_{x}=1-\prod_{r}\left[1-{}_{\frac{1}{12}}q_{x+r}\right]$$

These data should be suitably equalized by one of the equalization ordinary techniques known in literature, on which we don't dwell because considered not significant for the importance for the present work significance.
In the tables reported here below are first compared the probabilities $q_x$ and $\bar{q}_x$ acquired through the quantification of the people exposed to the risk or through the monthly rates analysis, and then the exit probabilities for reasons different from the death.






The two curves deviation is justified because of the approximation used: first of all, the curves acquired through the quantification of the Exposed to the Risk seem moved simply because with this methodology there is a data aggregation. In addition calculating the Exposed to the Risk have been adopted values approximations, which can be calculated monthly without having to calculate the people exposed to the risk.




In the above graph, it is reported the comparison among the actual rates for an individual, 57 years old, belonging to the collectivity under examination, to die within h years with $h = \frac{1}{12}, \frac{2}{12}, \dots, \frac{12}{12}$ and the frequencies acquired by applying the Balducci assumptions.
Besides, are reported the rates acquired by a further uniformity assumption [7] for which

$$ {}_{1-r}q_{x+r} = \frac{(1-r)q_x}{1 - rq_x} $$

## 6. Conclusion

In this work are reconsidered some results regarding the elimination pure and relative probabilities from a collectivity subject to more exit reasons, affording, in particular, the aspect related to the determination of these properties by raw historical data. The results acquired through the analysis of a real collectivity has highlighted as the exits annual probabilities for death or other reasons from a collectivity acquired through the quantification of the exposed to the risk result to be an approximation of the results acquired through a monthly analysis which describes better the collectivity trend. In fact, through this alternative technique, aren't necessary specific assumptions regarding the exit possibility (in particular on ${}_{1-r}q_{x+r}$) and the results acquired result effectively more adherent to the effective trend of the collectivity numerousness.

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Note:
[1] See TOMMASSETTI and OTHERS
[2] See MESSINA
[3] On the determination modality of the probabilities through the raw rates it is possible to watch Ottaviani, where are afforded in exhaustive manner the transition from the rates to the probability both in the case in which the event probability of which are known the rates remains steady in each following test, and in the case in which the probability changes according to a known or unknown law.
[4] This assumption is equivalent at a dotted linearity of the function $\frac{1}{S(x)}$ :

$$\frac{1}{S(x+r)} = (1-r)\frac{1}{S(x)} + r\frac{1}{S(x+1)}$$

$${}_{1-r}q_{x+r} = 1 - \frac{S(x+1)}{S(x+r)} = (1-r)q_x$$

[5] See PITACCO
[6] See PITACCO

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